

Resolving the puzzling decay patterns of charged Z_c and Z_b states

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We investigate the ratio of the branching fractions of the molecular candidates decaying into the ground and radially excited states within the quark interchange model. Our numerical results suggest that these molecular candidates are more likely to decay into the radially excited states than into ground states. Especially, the ratio $\Gamma[Z_c(4430) \rightarrow \pi\psi(2S)]/\Gamma[Z_c(4430) \rightarrow \pi J/\psi] \sim 9.8$ is close to the experimental measurement, which supports the interpretation of $Z_c(4430)$ as the $\bar{D}D^*(2S)$ molecular state. The ratios of the branching fractions of $Z_b(10610)$ and $Z_b(10650)$ to $\pi\Upsilon(2S, 3S)$ and $\pi\Upsilon(1S)$ agrees very well with the Belle's measurement. We also predict the similar ratios for $Z_c(3900)$, $Z_c(4020)$, $R_{Z_c(3900)} \approx 1.3$ and $R_{Z_c(4020)} \approx 4.7$. Hopefully the $\pi\psi(2S)$ mode, and ratios $R_{Z_c(3900)}$ and $R_{Z_c(4020)}$ will be measured by the BESIII and Belle collaborations in the near future, which shall be very helpful to understand the underlying dynamics of these exotic states.

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I. INTRODUCTION

The first evidence of the existence of charged charmonium-like states was reported by the Belle Collaboration in 2007 [1]. In $B \rightarrow K\pi^\pm\psi(2S)$ decays, a resonance-like structure $Z_c(4430)$ in the $\psi(2S)\pi^\pm$ mass distribution was observed. In 2013, the BESIII Collaboration observed $Z_c(3900)$ in the invariant mass spectrum of $J/\psi\pi^\pm$ from $Y(4260) \rightarrow J/\psi\pi\pi$ [2]. This observation was then quickly confirmed by the Belle Collaboration [3]. Another charged charmonium-like state $Z_c(4020)$ ($Z_c(4025)$) was observed by BESIII in the $h_c\pi^\pm$ and $(D^*\bar{D}^*)^\pm$ mass distributions [4–6]. There are also two charged bottomonium-like states $Z_b(10610)$ and $Z_b(10650)$ observed by Belle in the e^+e^- annihilation into the hidden-bottom dipion channels [7]. These states are of particular interest since their minimal quark contents are either $c\bar{c}d\bar{u}/c\bar{c}u\bar{d}$ or $b\bar{b}d\bar{u}/b\bar{b}u\bar{d}$, which are unambiguously beyond the conventional $q\bar{q}$ model.

Since its observation, $Z_c(4430)$ has attracted much attention both in experimental and theoretical research [8–18]. Because the mass of $Z_c(4430)$ is very close to the threshold of $D_1^{(\prime)}\bar{D}^*$, the interpretation of $Z_c(4430)$ as the loosely bound S-wave molecular state composed of $D_1^{(\prime)}\bar{D}^*$ had been quite popular among the various theoretical descriptions. Within this scheme, its possible spin-parity would be $J^P=0^-, 1^-$ or 2^- . However, after analyzing the $B^0 \rightarrow K^+\pi^-\psi(2S)$ decays, the LHCb Collaboration not only confirmed the existence of $Z_c(4430)$ but also determined its spin-parity to be $J^P=1^+$ unambiguously [19]. This measurement is obviously inconsistent with the $D_1^{(\prime)}\bar{D}^*$ molecular state theory, which leads to very puzzling new challenge in understanding the intrinsic structure of $Z_c(4430)$.

Very recently, it was proposed in Ref. [20] that $Z_c(4430)$ is the S-wave molecule composed of $\bar{D}D^*(2600)$ or $\bar{D}^*D(2550)$, where the dominant components of $D^*(2600)$ and $D(2550)$ are generally accepted as the radially excited states $D^*(2S)$ and $D(2S)$ respectively [21–23]. In other words, $Z_c(4430)$ may be the cousin of the charged states $Z_c(3900)$ and $Z_c(4020)$, which are speculated to be the molecular candidates composed of the D and D^* mesons. Several reasons lead to the above ansatz. Firstly, the thresholds of $\bar{D}D^*(2600)$ (~ 4477 MeV) and $\bar{D}^*D(2550)$ (~ 4546 MeV), especially the $\bar{D}D^*(2600)$ meson pair, are very close to the mass of $Z_c(4430)$, which is $M_{Z_c(4430)} = 4475 \pm 7_{-25}^{+15}$ MeV as measured by LHCb. Furthermore, the spin-parity of the S-wave $\bar{D}D^*(2600)$ combination is $J^P=1^+$, which is also consistent with the LHCb measurement. Since the width of $Z_c(4430)$ is very large ($\Gamma_{Z_c(4430)} = 172 \pm 13_{-34}^{+37}$ MeV [19]), which is much larger than the width of $D^*(2600)$ ($\Gamma_{D^*(2600)} = 93 \pm 6 \pm 13$ MeV [24]), such a molecular state assumption is still reasonable.

Another important motivation is due to the puzzling decay pattern of $Z_c(4430)$. It seems that $Z_c(4430)$ is more likely to decay

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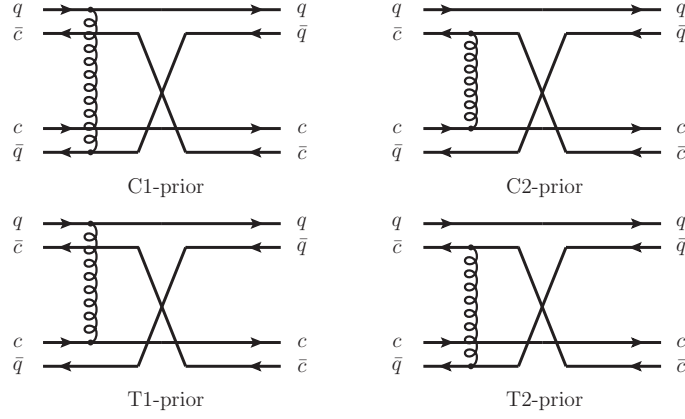


FIG. 1: Prior quark-interchange diagrams contributing to the anticharmed meson-charmed meson scattering into pion and charmonia.

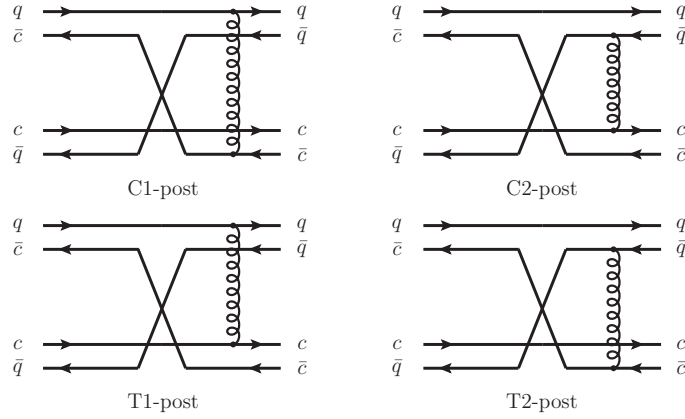


FIG. 2: Post quark-interchange diagrams contributing to the anticharmed meson-charmed meson scattering into pion and charmonia.

into $\psi(2S)\pi$ than into $J/\psi\pi$, i.e., the ratio $Br(Z_c(4430)^+ \rightarrow \psi(2S)\pi^+)/Br(Z_c(4430)^+ \rightarrow J/\psi\pi^+)$ is about 10 as measured by Belle [25]. If $Z_c(4430)$ contains $D(2S)$ or $D^*(2S)$ as its component, one would expect that it will decay into the final state containing a radial excitation easily, i.e., the $\psi(2S)\pi$ channel may be its favorable decay mode.

In this paper we will investigate the difference between the $J/\psi\pi$ and $\psi(2S)\pi$ decay channels of $Z_c(4430)$, $Z_c(3900)$ and $Z_c(4020)$. The calculation of the pertinent branching ratios is mainly based on a non-relativistic quark-interchange model. The $\Upsilon(nS)\pi$ decay modes of the charged bottomnium-like states $Z_b(10610)$ and $Z_b(10650)$ will also be studied.

II. THE MODEL

For the loosely bound S-wave molecular candidates $Z_c(4430)$, $Z_c(3900)$ and $Z_c(4020)$ decaying into $J/\psi\pi$ and $\psi(2S)\pi$, we can describe these decay processes as the anticharmed meson-charmed meson scattering processes near the threshold. In these reactions, c and \bar{c} are recombined into a charmonium state, which is governed by the short range interaction. To describe these meson-meson scatterings at the quark level, we will employ the Barnes-Swanson quark-interchange model to estimate the transition amplitudes [26–31]. In this approach, the non-relativistic quark potential model is used, and the hadron-hadron scattering amplitudes are evaluated at Born order with the interquark Hamiltonian. In the case of the anticharmed meson-charmed meson scatterings, the amplitudes arise from the sum of the four quark exchange diagrams as illustrated in Fig. 1.

The interaction H_{ij} between constituents i and j is represented by the curly line in Fig. 1, and is taken to be

$$\begin{aligned}
 H_{ij} &\equiv \frac{\lambda(i)}{2} \cdot \frac{\lambda(j)}{2} V_{ij}(r_{ij}) = \frac{\lambda(i)}{2} \cdot \frac{\lambda(j)}{2} (V_{conf} + V_{hyp} + V_{con}) \\
 &= \frac{\lambda(i)}{2} \cdot \frac{\lambda(j)}{2} \left\{ \frac{\alpha_s}{r_{ij}} - \frac{3b}{4} r_{ij} - \frac{8\pi\alpha_s}{3m_i m_j} \mathbf{S}_i \cdot \mathbf{S}_j \left(\frac{\sigma^3}{\pi^{3/2}} \right) e^{-\sigma^2 r_{ij}^2} + V_{con} \right\},
 \end{aligned} \tag{1}$$

where for antiquarks the color Gell-Mann matrix λ should be replaced by $-\lambda^T$. This Hamiltonian contains a Coulomb-plus-

linear confining potential V_{conf} and a short range spin-spin hyperfine term V_{hyp} , which is motivated by one gluon exchange. The model parameters employed here are $m_u=m_d=0.334$ GeV, $m_c=1.776$ GeV, $m_b=5.102$ GeV, $\sigma=0.897$ GeV, $b=0.18$ GeV², and the constant V_{con} is taken to be 0.62 GeV [30]. We use a running coupling constant $\alpha_s(Q^2)$, which is given by

$$\alpha_s(Q^2) = \frac{12\pi}{(33 - 2n_f)\ln(A + Q^2/B^2)} \quad (2)$$

with $A=10$, $B=0.31$ GeV. And the scale Q^2 is identified with the square of the invariant mass of the interacting constituents. These conventional quark model parameters lead to a good description of the meson spectrum, as listed in Table I.

TABLE I: Estimated meson masses according to the quark model.

	D	D^*	$D(2S)$	$D^*(2S)$	J/ψ	$\psi(2S)$	B	B^*	B_1	$\Upsilon(1S)$	$\Upsilon(2S)$	$\Upsilon(3S)$
M_{th} [GeV]	1.920	1.993	2.711	2.769	3.089	3.701	5.387	5.411	5.748	9.471	9.944	10.347
M_{exp} [GeV]	1.870	2.010	2.539	2.612	3.097	3.686	5.279	5.325	5.724	9.460	10.023	10.355

Following Refs. [26–31], we can write out the Born-order T -matrix element T_{fi} which is the product of three factors for each of the diagrams in Fig. 1, i.e.,

$$T_{fi} = (2\pi)^3 I_{flavor} I_{color} I_{spin-space}. \quad (3)$$

Since there is no orbitally excited state involved in our discussion, the factor $I_{spin-space}$ can be further factored into

$$I_{spin-space} = I_{spin} \times I_{space}. \quad (4)$$

For the processes discussed in this paper, all of the quarks are distinguishable, and we assume the external flavor states all have a positive phase. The flavor factor I_{flavor} is then simply unity. For the color factor I_{color} , the diagrams C1 and C2 of Fig. 1 have factors of $-4/9$, and the diagrams T1 and T2 have factors of $+4/9$. We list the spin factors for the operator $\mathbf{S}_i \cdot \mathbf{S}_j$ of each diagram in Table II. The spin factors of the unit operator for all of the diagrams are equal, as listed in Table III.

TABLE II: Spin factors I_{spin} for the operator $\mathbf{S}_i \cdot \mathbf{S}_j$ of each diagram. Here, S_A (S_B , S_C , S_D) is the spin of meson A (B , C , D) in the reaction $AB \rightarrow CD$.

$(S_A, S_B) \rightarrow (S_C, S_D)$	C1 prior	C1 post	C2 prior	C2 post	T1 prior	T1 post	T2 prior	T2 post
$(0, 1) \rightarrow (0, 1)$	$-\frac{3}{8}$	$-\frac{3}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$
$(1, 1) \rightarrow (0, 1)$	$-\frac{3}{4\sqrt{2}}$	$\frac{1}{4\sqrt{2}}$	$\frac{1}{4\sqrt{2}}$	$\frac{1}{4\sqrt{2}}$	$-\frac{1}{4\sqrt{2}}$	$-\frac{1}{4\sqrt{2}}$	$-\frac{1}{4\sqrt{2}}$	$-\frac{1}{4\sqrt{2}}$

TABLE III: Spin factors I_{spin} for the unit operator.

$(S_A, S_B) \rightarrow (S_C, S_D)$	All diagrams
$(0, 1) \rightarrow (0, 1)$	$\frac{1}{2}$
$(1, 1) \rightarrow (0, 1)$	$\frac{1}{\sqrt{2}}$

The space factors are evaluated as the overlap integrals of the quark model wave functions. It is convenient to write these overlap integrals in the momentum-space. For the diagrams of Fig. 1, in the reaction $AB \rightarrow CD$, where AB and CD are the initial and final meson pairs, respectively. The space factors read as

$$I_{space}^{C1-prior} = \int \int d\mathbf{k} d\mathbf{q} \Phi_A(2\mathbf{k}) \Phi_B(2\mathbf{k} - 2\mathbf{P}_C) \Phi_C(2\mathbf{q} - \mathbf{P}_C) \Phi_D(2\mathbf{k} - \mathbf{P}_C) V(\mathbf{k} - \mathbf{q}), \quad (5)$$

$$I_{space}^{C2-prior} = \int \int d\mathbf{k} d\mathbf{q} \Phi_A(-2\mathbf{k}) \Phi_B(-2\mathbf{k} - 2\mathbf{P}_C) \Phi_C(-2\mathbf{k} - \mathbf{P}_C) \Phi_D(-2\mathbf{q} - \mathbf{P}_C) V(\mathbf{k} - \mathbf{q}), \quad (6)$$

$$I_{space}^{T1} = \int \int d\mathbf{k} d\mathbf{q} \Phi_A(2\mathbf{k}) \Phi_B(2\mathbf{q} - 2\mathbf{P}_C) \Phi_C(2\mathbf{q} - \mathbf{P}_C) \Phi_D(2\mathbf{k} - \mathbf{P}_C) V(\mathbf{k} - \mathbf{q}), \quad (7)$$

$$I_{space}^{T2} = \int \int d\mathbf{k} d\mathbf{q} \Phi_A(-2\mathbf{k}) \Phi_B(-2\mathbf{q} - 2\mathbf{P}_C) \Phi_C(-2\mathbf{k} - \mathbf{P}_C) \Phi_D(-2\mathbf{q} - \mathbf{P}_C) V(\mathbf{k} - \mathbf{q}), \quad (8)$$

where \mathbf{P}_C is the center-of mass momentum of meson C . And the potential $V(\mathbf{p})$ is obtained via the Fourier transformation of $V(r)$. Taking into account that, we are actually estimating the decaying amplitudes of the molecular states, of which the components are A and B . Therefore, the center-of-mass momenta of A and B can be approximately taken as $\mathbf{P}_A = -\mathbf{P}_B \approx 0$. We have used these approximations in deriving the above equations.

To simplify the spatial overlap integrals, we expand the wave functions $\Psi(\mathbf{r})$ in the coordinate space of each state as a linear combination of Gaussian basis functions [30], i.e.

$$\Psi(\mathbf{r}) = \sum_{n=1}^N a_n \frac{(n\beta^2)^{3/4}}{\pi^{3/4}} e^{-n\beta^2 \mathbf{r}^2/2}, \quad (9)$$

where β denotes the width parameter of Gaussian functions. a_n is the expansion coefficient, and we take $N=6$ in our calculations. Using the interaction in Eq. (1) and solving the eigenvalue equation, we obtain the mass spectrum and the corresponding wave functions which are displayed in Table I and Fig. 3, respectively. To fit the pertinent hadron mass spectrum well, we set the width parameter β as 0.3 GeV and 0.61 GeV for charmed mesons and bottomed mesons, respectively, which is different from the fitting parameters in Ref [30]. Since the wave functions of the radially excited states have nodes, the overlap integrals for $\pi J/\psi$ and $\pi\psi(2S)$ final states are very different. In other words, the branching fractions of $Z_c(4430)$ ($Z_c(3900)$, and $Z_c(4020)$) decaying into $\pi J/\psi$ and $\pi\psi(2S)$ will be different.

There is still a "prior-post" ambiguity while calculating the scattering amplitudes via the quark-interchange model [26–31]. The Hamiltonian that describes the $AB \rightarrow CD$ process can be separated into free and interaction parts in two ways, i.e. $H = H_A^0 + H_B^0 + V_{AB}$ or $H = H_C^0 + H_D^0 + V_{CD}$. In the "prior" scattering diagrams of Fig. 1, the interactions occur before the quark interchange, and the interaction Hamiltonian is taken as V_{AB} . There are also some corresponding "post" diagrams displayed in Fig. 2, where the interactions occur after the quark interchange, and the interaction Hamiltonian is V_{CD} . The spin factors for the post diagrams are also listed in Tables II and III. The space factors for T_1 and T_2 are the same as those of prior diagrams, and the factors for C1-post and C2-post read as

$$I_{space}^{C1-post} = \int \int d\mathbf{k} d\mathbf{q} \Phi_A(2\mathbf{k}) \Phi_B(2\mathbf{q} - 2\mathbf{P}_C) \Phi_C(2\mathbf{q} - \mathbf{P}_C) \Phi_D(2\mathbf{q} - \mathbf{P}_C) V(\mathbf{k} - \mathbf{q}), \quad (10)$$

$$I_{space}^{C2-post} = \int \int d\mathbf{k} d\mathbf{q} \Phi_A(-2\mathbf{k}) \Phi_B(-2\mathbf{q} - 2\mathbf{P}_C) \Phi_C(-2\mathbf{k} - \mathbf{P}_C) \Phi_D(-2\mathbf{k} - \mathbf{P}_C) V(\mathbf{k} - \mathbf{q}). \quad (11)$$

If one uses the exact bound state wave functions determined by the full Hamiltonian, and the interaction used in calculating the scattering amplitudes is identical to that used in calculating the wave functions, the prior and post scattering amplitudes should be equal. However, the numerical test of the prior-post equivalence shows some relatively large sensitivities to the wave functions [26–31].

In the present case, this ambiguity can be greatly removed based on the physical considerations. According to the OZI rule, the interaction between the charmed meson and anti-charmed meson should be much stronger than that between the light hadron and charmonium state, which was confirmed by the lattice QCD simulation [32]. Taking into account this point, we will mainly discuss the scattering process according to the "prior" diagrams.

III. NUMERICAL RESULTS

In this work, we focus on the difference between the branching fractions of Z_c^\pm (Z_b^\pm) decaying into $J/\psi\pi^\pm$ ($\Upsilon(1S)\pi^\pm$) and $\psi(2S)\pi^\pm$ ($\Upsilon(2S, 3S)\pi^\pm$), and ignore the dynamics of the heavy meson pairs forming the molecular states for the moment. We introduce the ratios of the branching fractions of the XYZ state to the radially excited and ground states as follows:

$$R_{Z_c} \equiv \frac{\Gamma[Z_c \rightarrow \pi\psi(2S)]}{\Gamma[Z_c \rightarrow \pi J/\psi]}, \quad R_{Z_b} \equiv \frac{\Gamma[Z_b \rightarrow \pi\Upsilon(2S)]}{\Gamma[Z_b \rightarrow \pi\Upsilon(1S)]}, \quad R'_{Z_b} \equiv \frac{\Gamma[Z_b \rightarrow \pi\Upsilon(3S)]}{\Gamma[Z_b \rightarrow \pi\Upsilon(1S)]}, \quad (12)$$

where Z_c could be $Z_c(4430)$, $Z_c(3900)$ and $Z_c(4020)$ respectively, and Z_b could be $Z_b(10610)$ and $Z_b(10650)$ respectively. Since the measured masses of these molecular candidates still have relatively large uncertainties, we will first give the mass dependence of the ratios in Eq. (12). The numerical results are displayed in Figs. 4 and 5. Apart from $R_{Z_c(4430)}$, the other ratios are not sensitive to the masses of the corresponding molecular states.

Some of these molecular candidates have large width. For instance, the width of $Z_c(4430)$ is about 172 MeV [19]. One therefore has to take into account the mass distribution of these states while investigating their decay channels. Their two-body decay width can be calculated as:

$$\Gamma[Z_c]_{2\text{-body}} = \frac{1}{W} \int_{s_{th}}^{(m_{Z_c} + 2\Gamma_{Z_c})^2} ds \frac{(2\pi)^4}{2\sqrt{s}} \int d\Phi_2 |\mathcal{M}_{fi}|^2 \frac{1}{\pi} \text{Im} \left(\frac{-1}{s - m_{Z_c}^2 + im_{Z_c}\Gamma_{Z_c}} \right), \quad (13)$$

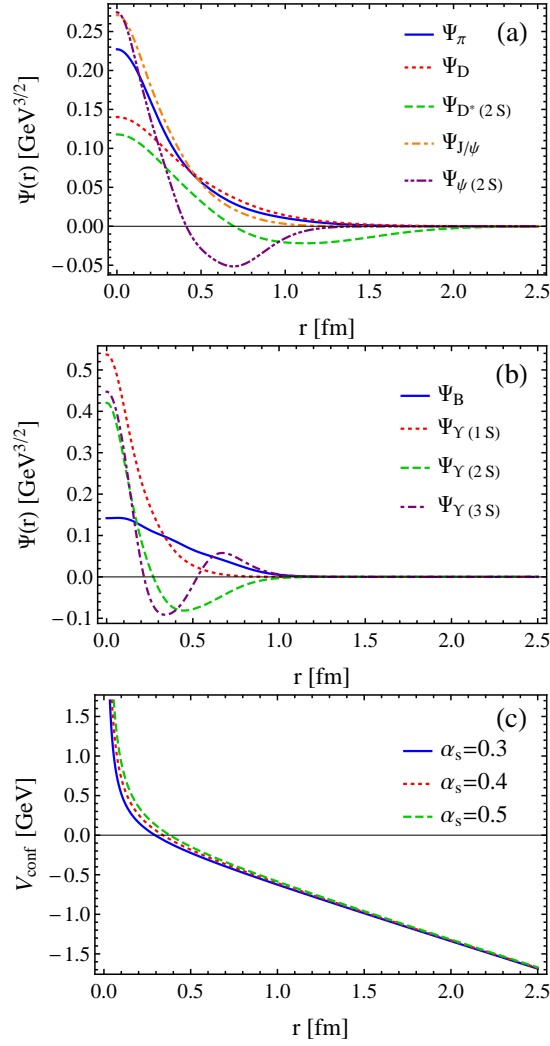


FIG. 3: (a) Wave functions of π , D , $D^*(2S)$, J/ψ and $\psi(2S)$ in coordinate space. (b) Wave functions of B , $\Upsilon(1S)$, $\Upsilon(2S)$ and $\Upsilon(3S)$ in coordinate space. (c) the r -dependence of the confining potential $V_{\text{conf}} = \frac{\alpha_s}{r} - \frac{3b}{4}r$ with $b=0.18 \text{ GeV}^2$.

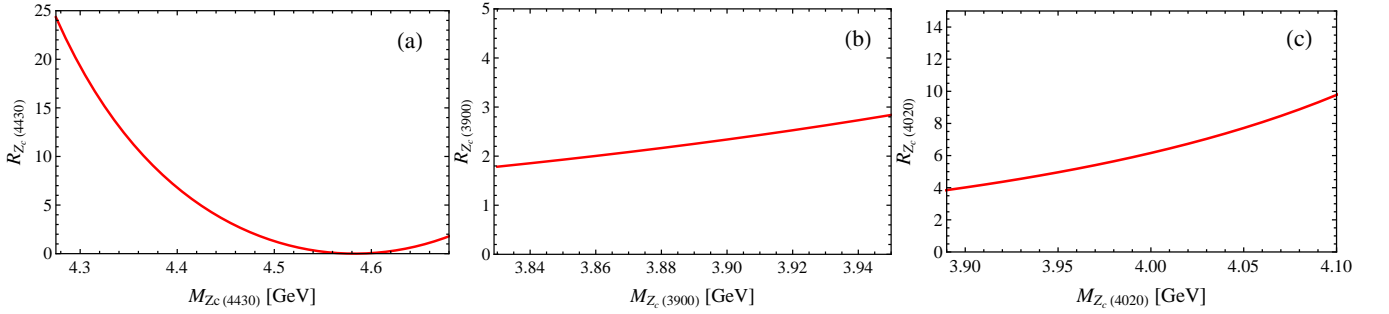


FIG. 4: The mass-dependence of the ratios $R_{Z_c(4430)}$, $R_{Z_c(3900)}$ and $R_{Z_c(4020)}$.

where $d\Phi_2$ is the two-body phase space. s_{th} is the energy threshold, and we set s_{th} as $(m_{\psi(2S)} + m_\pi)^2$ and $(m_{\Upsilon(3S)} + m_\pi)^2$ for Z_c and Z_b decays respectively. W is a normalization factor, which reads as

$$W = \frac{1}{\pi} \int_{s_{th}}^{(m_{Z_c} + 2\Gamma_{Z_c})^2} ds \text{Im} \left(\frac{-1}{s - m_{Z_c}^2 + im_{Z_c}\Gamma_{Z_c}} \right). \quad (14)$$

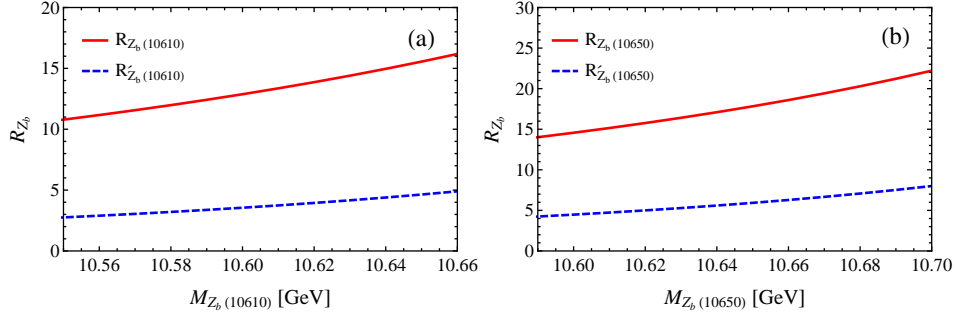


FIG. 5: The mass-dependence of the ratios $R_{Z_b(10610)}$, $R'_{Z_b(10610)}$, $R_{Z_b(10650)}$ and $R'_{Z_b(10650)}$.

\mathcal{M}_{fi} is related to T_{fi} by

$$\mathcal{M}_{fi} = \sqrt{E_A E_B E_C E_D} T_{fi}, \quad (15)$$

where E_A , E_B , E_C and E_D are the energies of meson A , B , C and D in the center-of-mass system, respectively, for $AB \rightarrow CD$ reaction. The obtained ratios are listed in Table IV, where we have used the center values of experimental data for M_{Z_c} , M_{Z_b} , Γ_{Z_c} and Γ_{Z_b} .

Assuming that the components of $Z_c(4430)$ are composed of $\bar{D}D^*(2S)$ [20], we obtain $R_{Z_c(4430)} \approx 9.8$, which is very close to the experimental data. The nodes in the wave functions of $D^*(2S)$ and $\psi(2S)$ ensure the larger spatial overlap integral and render $\pi\psi(2S)$ to be a more favorable mode of $Z_c(4430)$.

If $Z_c(3900)$ and $Z_c(4020)$ are the $\bar{D}D^*$ and \bar{D}^*D^* molecular states, respectively, [34], we derive the ratios $R_{Z_c(3900)} \approx 1.3$ and $R_{Z_c(4020)} \approx 4.7$. According to the above results, it seems that these molecular states are more likely to decay into $\pi\psi(2S)$ than into $\pi J/\psi$. This conclusion can be qualitatively understood according to the node positions of the wave functions and potentials. In Fig. 3, we also display the r -dependence of the confining potential V_{con} , which crosses zero around 0.4 fm. The wave function of $\psi(2S)$ also has a node around 0.4 fm while the wave function of J/ψ does not have a node. Recall that the wave functions of the initial states are the same and do not have nodes. Therefore the overlap integral for $\pi J/\psi$ final states is much smaller than that for $\pi\psi(2S)$ final states. Notice that $R_{Z_c(4020)}$ is much larger than 1, which implies that it is very hopeful to find $Z_c(4020)$ in the final states containing $\pi\psi(2S)$, such as the reaction $e^+e^- \rightarrow \psi(2S)\pi\pi$.

Similarly, we assume $Z_b(10610)$ and $Z_b(10650)$ to be molecular states composed of $B\bar{B}^*$ and $B^*\bar{B}^*$ respectively [35]. As displayed in Table IV, the estimated ratios for $Z_b(10610)$ and $Z_b(10650)$ are also very close to the experimental measurements by the Belle Collaboration, which favors the $B\bar{B}^*$ and $B^*\bar{B}^*$ molecular states explanations concerning the Z_b states.

Although some results which are consistent with the data are obtained, we should mention that there are still some theoretical uncertainties within the present framework. One major uncertainty arises from the hadron wave functions, because it may not be a very good approximation to treat the pion meson and open flavor mesons as the non-relativistic systems. However, the potentials and wave functions do not vary too much among different quark models. Our qualitative estimation of the overlap integrals mentioned in the last several paragraphs still works, which will be a less model-dependent conclusion.

TABLE IV: The ratios after taking into account the width of the molecular candidate. Here, the experimental ratios are estimated according to Refs. [25, 33], and we only take into account the statistical error of the data.

	$R_{Z_c(4430)}$	$R_{Z_c(3900)}$	$R_{Z_c(4020)}$	$R_{Z_b(10610)}$	$R'_{Z_b(10610)}$	$R_{Z_b(10650)}$	$R'_{Z_b(10650)}$
Theo.	9.8	1.3	4.7	11.4	2.2	15.8	4.0
Exp.	~ 10	\dots	\dots	$4.8 \sim 8.8$	$2.7 \sim 5.3$	$4.5 \sim 12.1$	$5.4 \sim 14.0$

IV. SUMMARY

In this work, we have systematically discussed the puzzling decay properties of the exotic states $Z_c(3900)$, $Z_c(4020)$, and their radially excited cousin $Z_c(4430)$. With the quark-interchange model, we have derived the ratios of the branching fractions of these molecular candidates decaying into the ground states and radially excited states. Our numerical results imply that these molecular candidates are more likely to decay into radially excited states than into ground states.

As a $\bar{D}D^*(2S)$ molecular candidate, $R_{Z_c(4430)}$ tends to decay into $\pi\psi(2S)$ more easily. The interplay of the node in the wave function of $D^*(2S)$ and the node in the wave function of $\psi(2S)$ ensures the larger spatial overlap integral of the scattering amplitude. We obtain the ratio $R_{Z_c(4430)} \sim 9.8$, which is close to the experimental measurement. In fact, the measured ratio favors the $\bar{D}D^*(2S)$ molecular state ansatz concerning the intrinsic structure of $Z_c(4430)$. The above decay pattern is very characteristic for a molecular state containing a radially excited component.

In contrast, if $Z_c(4430)$ is an S-wave tetraquark ground state, it will be extremely challenging to accommodate the experimental ratio $R_{Z_c(4430)} \sim 10$. If $Z_c(4430)$ is a radially excited tetraquark state, one may also expect $\pi\psi(2S)$ to be a more favorable decay mode. Then, where is the ground state tetraquark?

We have also estimated the ratios for $Z_b(10610)$ and $Z_b(10650)$ assuming that they are molecular candidates. Our numerical results show that the favorable decay modes of $Z_b(10610)$ and $Z_b(10650)$ are $\pi\Upsilon(2S)$ and $\pi\Upsilon(3S)$, which is consistent with the experimental measurement by the Belle Collaboration.

Assuming $Z_c(3900)$ and $Z_c(4020)$ are the $\bar{D}D^*$ and \bar{D}^*D^* molecular states, we have predicted the ratios $R_{Z_c(3900)} \approx 1.3$ and $R_{Z_c(4020)} \approx 4.7$. The accidental match of the node of the interaction potential with the node of the wave function of $\psi(2S)$ enhances the spatial overlap integral of the scattering amplitude and renders $\pi\psi(2S)$ to be a favorable decay mode. Hopefully the $\pi\psi(2S)$ mode, and ratios $R_{Z_c(3900)}$ and $R_{Z_c(4020)}$ will be measured by the BESIII and Belle collaborations in the near future, which shall be very helpful to understand the underlying dynamics of these exotic states.

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